

Anisotropic Dark Energy Bianchi Type *III* Cosmological Models in Brans Dicke Theory of Gravity

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Abstract

The main purpose of this paper is to explore the solutions of Bianchi type *III* cosmological model in Brans Dicke theory of gravity in the background of anisotropic dark energy. We use the assumption of constant deceleration parameter and power law relation between scalar field ϕ and scale factor a to find the solutions. The physical behavior of the solutions has been discussed using some physical quantities.

Keywords: Bianchi type *III*, Dark energy and Brans Dicke theory of gravity.

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1 Introduction

Recent experimental data [1] about late time accelerated expansion of the universe has attracted much attention in the recent years. Cosmic acceleration can be well explained from high red-shift supernova experiments . The

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recent results from cosmic microwave background fluctuations [2] and large scale structure [3] suggest the expansion of universe. Dark energy seems to be best candidate to explain cosmic acceleration. It is now believed that 96 percent energy of the universe consist of dark energy and dark matter (76 percent dark energy and 20 percent dark matter) [2, 4]. Dark energy is the most popular way to explain recent observations that the universe is expanding at an accelerating rate. The exact nature of the dark energy is a matter of speculation. It is known to be very homogeneous, not very dense and is not known to interact through any of the fundamental forces other than gravity. Since it is not very dense, roughly $10 - 29$ grams per cubic centimeter, so it is a difficult task to detect it in the laboratory. It is thought that dark energy have a strong negative pressure in order to explain the observed acceleration in the expansion rate of the universe.

Dark energy models have significant importance now as far as theoretical study of the universe is concerned. It would be more interesting to study the variable equation of state (EoS), i.e. $P = \rho\omega(t)$, where P is the pressure and ρ is the energy density of universe. Usually EoS parameter is assumed to be a constant with the values -1 , 0 , $-\frac{1}{3}$ and $+1$ for vacuum, dust, radiation and stiff matter dominated universe, respectively. However, it is a function of time or redshift [5] in general. Latest observations [6] from SNe Ia data indicate that ω is not constant. In recent years, many authors [7]-[11] have shown keen interests in studying the universe with variable EoS. Sharif and Zubair [7] discussed the dynamics of Bianchi type VI_0 universe with anisotropic dark energy in the presence of electromagnetic field. The same authors [8] explored Bianchi type I universe in the presence of magnetized anisotropic dark energy with variable EoS parameter. Akarsu and Kilinc [9] investigated the general form of the anisotropy parameter of the expansion for Bianchi type III model.

The isotropic models are considered to be most suitable to study large scale structure of the universe. However, it is believed that the early universe may not have been exactly uniform. This prediction motivates us to describe the early stages of the universe with the models having anisotropic background. Thus, it would be worthwhile to explore anisotropic dark energy models in the context of modified theories of gravity. Among the various modifications of general relativity (GR), the Brans-Dicke (BD) theory of gravity [12] is a well known example of a scalar tensor theory in which the gravitational interaction involves a scalar field and the metric tensor. One extra parameter ϖ is used in this theory which satisfies the equation given

by

$$\square\phi = \frac{8\pi T}{3 + 2\varpi},$$

where ϕ is known as BD scalar field while T is the trace of the matter energy-momentum tensor. It is mentioned here that the general relativity is recovered in the limiting case $\varpi \rightarrow \infty$. Thus we can compare our results with experimental tests for significantly large value of ϖ .

Bianchi type models are among the simplest models with anisotropic background. Many authors [13]-[24] explored Bianchi type spacetimes in different contexts. Moussiaux et al. [25] investigated the exact solution for vacuum Bianchi type-*III* model with a cosmological constant. Xing-Xiang [26] discussed Bianchi type *III* string cosmology with bulk viscosity. He assumed that the expansion scalar is proportional to the shear scalar to find the solutions. Wang [27] investigated string cosmological models with bulk viscosity in Kantowski-Sachs spacetime. Upadhaya [28] explored some magnetized Bianchi type-*III* massive string cosmological models in GR. Hellaby [29] gave an overview of some recent developments in inhomogeneous models and it was concluded that the universe is inhomogeneous on many scales. Sharif and Shamir [17, 18] have studied the solutions of Bianchi types *I* and *V* spacetimes in the framework of $f(R)$ gravity. Recently, we [19, 20] explored the exact vacuum solutions of Bianchi type *I*, *III* and Kantowski-Sachs spacetimes in the metric version of $f(R)$ gravity.

The study of Bianchi type models in the context of BD theory has attracted many authors in the recent years [30]. A detailed discussion of BD cosmology is given by Singh and Rai [31]. Lorenz-Petzold [32] studied exact Bianchi type-*III* solutions in the presence of electromagnetic field. Kumar and Singh [33] investigated perfect fluid solutions using Bianchi type *I* spacetime in scalar-tensor theory. Adhav et al. [34] obtained an exact solution of the vacuum Brans-Dicke field equations for the metric tensor of a spatially homogeneous and anisotropic model. Pradhan and Amirhashchi [35] investigated anisotropic dark energy Bianchi type *III* model with variable EoS parameter in GR. Adhav et al. [36] explored Bianchi type *III* cosmological model with negative constant deceleration parameter in Brans Dicke theory of gravity in the presence of perfect fluid.

In this paper, we focuss our attention to explore the solutions of anisotropic dark energy Bianchi type *III* cosmological model in the context of BD theory of gravity. We find the solutions using the assumption of constant deceleration parameter and power law relation between ϕ and a . The paper is orga-

nized as follows: A brief introduction of the field equations in BD theory of gravity is given in section 2. In section 3, the solutions of the field equations for Bianchi types *III* spacetime are found. Some physical and kinematical parameters are also evaluated for the solutions. Singularity analysis is given in section 4. A brief summary is given in the last section.

2 Some Basics of Brans Dicke theory of Gravity

The line element for the spatially homogeneous and anisotropic Bianchi type *III* spacetime is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{-2mx}B^2(t)dy^2 - C^2(t)dz^2, \quad (1)$$

where A , B and C are cosmic scale factors and m is a positive constant. The energy momentum tensor for anisotropic dark energy is given by

$$T^i_j = \text{diag}[\rho, -p_x, -p_y, -p_z] = \text{diag}[1, -\omega_x, -\omega_y, -\omega_z]\rho, \quad (2)$$

where ρ is the energy density of the fluid while p_x , p_y and p_z are the pressures on the x , y and z axes respectively. Here ω is EoS parameter of the fluid with no deviation and ω_x , ω_y and ω_z are the EoS parameters in the directions of x , y and z axes respectively. The energy momentum tensor can be parameterized as

$$T^i_j = \text{diag}[1, -\omega, -(\omega + \gamma), -(\omega + \delta)]\rho. \quad (3)$$

For the sake of simplicity, we choose $\omega_x = \omega$ and the skewness parameters γ and δ are the deviations from ω on y and z axes respectively.

The Brans-Dicke field equations are

$$R_{ij} - \frac{1}{2}Rg_{ij} - \frac{\varpi}{\phi^2}(\phi_{;i}\phi_{;j} - \frac{1}{2}g_{ij}\phi_{;k}\phi^{;k}) - \frac{1}{\phi}(\phi_{;ij} - g_{ij}\square\phi) = \frac{8\pi T_{ij}}{\phi}, \quad (4)$$

and

$$\square\phi = \phi^{;k}_{;k} = \frac{8\pi T}{3 + 2\varpi}, \quad (5)$$

where ϖ is a dimensionless coupling constant. For Bianchi type *III* space-time, the field equations take the form

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{m^2}{A^2} - \frac{\varpi}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{8\pi\rho}{\phi}, \quad (6)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{\ddot{\phi}}{\phi} + \frac{\varpi}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = -\frac{8\pi\omega\rho}{\phi}, \quad (7)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} + \frac{\ddot{\phi}}{\phi} + \frac{\varpi}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\right) = -\frac{8\pi(\omega + \gamma)\rho}{\phi}, \quad (8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{m^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{\phi}}{\phi} + \frac{\varpi}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = -\frac{8\pi(\omega + \delta)\rho}{\phi}. \quad (9)$$

Also, the 01-component can be written in the following form

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0. \quad (10)$$

Integrating this equation, we obtain

$$B = c_1 A, \quad (11)$$

where c_1 is an integration constant. Without loss of any generality, we take $c_1 = 1$. Using Eq.(5), we get

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{8\pi(1 - 3\omega - \delta - \gamma)\rho}{\phi(3 + 2\varpi)}. \quad (12)$$

Now we define some physical parameters before solving the field equations.

The average scale factor a and the volume scale factor V are defined as

$$a = \sqrt[3]{A^2 C}, \quad V = a^3 = A^2 C. \quad (13)$$

The generalized mean Hubble parameter H is given in the form

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \quad (14)$$

where $H_1 = \frac{\dot{A}}{A} = H_2$, $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions of x , y and z axis respectively. Using Eqs.(13) and (14), we obtain

$$H = \frac{1}{3} \frac{\dot{V}}{V} = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{\dot{a}}{a}. \quad (15)$$

The expansion scalar θ and shear scalar σ are defined as follows

$$\theta = u^\mu_{;\mu} = 2\frac{\dot{A}}{A} + \frac{\dot{C}}{C}, \quad (16)$$

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} = \frac{1}{3}\left[\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right]^2, \quad (17)$$

where

$$\sigma_{\mu\nu} = \frac{1}{2}(u_{\mu;\alpha}h_\nu^\alpha + u_{\nu;\alpha}h_\mu^\alpha) - \frac{1}{3}\theta h_{\mu\nu}, \quad (18)$$

$h_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$ is the projection tensor while $u_\mu = \sqrt{g_{00}}(1, 0, 0, 0)$ is the four-velocity in co-moving coordinates. The mean anisotropy parameter A_m is defined as

$$A_m = \frac{1}{3} \sum \left(\frac{\Delta H_i}{H}\right)^2, \quad (19)$$

where $\Delta H_i = H_i - H$, ($i = 1, 2, 3$).

3 Solution of the Field Equations

Subtracting Eq.(7) and Eq.(8), we get

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{C}}{C}\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right) + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right) = \frac{8\pi\gamma\rho}{\phi}. \quad (20)$$

Using Eq.(11), this equation gives $\gamma = 0$. Thus Eqs.(6-9) and Eq.(12) reduce to

$$\left(\frac{\dot{A}}{A}\right)^2 + \frac{2\dot{C}\dot{A}}{CA} - \frac{m^2}{A^2} - \frac{\varpi}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{\phi}}{\phi}\left(2\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) = \frac{8\pi\rho}{\phi}, \quad (21)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} + \frac{\ddot{\phi}}{\phi} + \frac{\varpi}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\right) = -\frac{8\pi\omega\rho}{\phi}, \quad (22)$$

$$\frac{2\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 - \frac{m^2}{A^2} + \frac{\ddot{\phi}}{\phi} + \frac{\varpi}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{2\dot{\phi}\dot{A}}{\phi A} = -\frac{8\pi(\omega + \delta)\rho}{\phi}, \quad (23)$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{2\dot{A}}{A} + \frac{\dot{C}}{C}\right) = \frac{8\pi(1 - 3\omega - \delta)\rho}{\phi(3 + 2\varpi)}. \quad (24)$$

Integration after subtracting Eq.(22) and Eq.(23) yields

$$\frac{\dot{A}}{A} - \frac{\dot{C}}{C} = \frac{1}{A^2 C \phi} \int \left(\frac{m^2}{A^2} - \frac{8\pi\delta\rho}{\phi}\right) \phi A^2 C \, dt + \frac{\lambda}{A^2 C \phi}, \quad (25)$$

where λ is an integration constant. The integral term in this equation vanishes for

$$\delta = \frac{m^2 \phi}{8\pi\rho A^2}. \quad (26)$$

Using Eq.(26) in Eq.(25), it follows that

$$\frac{A}{C} = c_2 e^{\lambda \int \frac{dt}{a^3 \phi}}, \quad (27)$$

where $a^3 = A^2 C$ and c_2 is an integration constant. Here we use the power law assumption to solve the integral part in the above equations. The power law relation between scale factor a and scalar field ϕ has already been used by Johri and Desikan [37] in the context of Robertson Walker Brans-Dicke models. Thus the power law relation between ϕ and a , i.e. $\phi \propto a^m$, where m is any integer, implies that

$$\phi = b a^m, \quad (28)$$

where b is the constant of proportionality. The deceleration parameter q in cosmology is the measure of the cosmic acceleration of the universe expansion and is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2}. \quad (29)$$

It is mentioned here that q was supposed to be positive initially but recent observations from the supernova experiments suggest that it is negative. Thus the behavior of the universe models depend upon the sign of q . The positive deceleration parameter corresponds to a decelerating model while the negative value provides inflation. We also use a well-known relation [38] between the average Hubble parameter H and average scale factor a given as

$$H = l a^{-n}, \quad (30)$$

where $l > 0$ and $n \geq 0$. This is an important relation because it gives the constant value of the deceleration parameter. From Eq.(15) and Eq.(30), we get

$$\dot{a} = l a^{-n+1}. \quad (31)$$

Using this value, we find that deceleration parameter is constant, i.e. $q = n - 1$. Integrating Eq.(31), it follows that

$$a = (nlt + k_1)^{\frac{1}{n}}, \quad n \neq 0 \quad (32)$$

and

$$a = k_2 \exp(lt), \quad n = 0, \quad (33)$$

where k_1 and k_2 are constants of integration. Thus we obtain two values of the average scale factor that correspond to two different models of the universe.

3.1 Dark Energy Model of the Universe when $n \neq 0$.

Now we discuss the model of universe when $n \neq 0$, i.e., $a = (nlt + k_1)^{\frac{1}{n}}$. For this model, ϕ becomes

$$\phi = b(nlt + k_1)^{-\frac{2}{n}}. \quad (34)$$

Using this value of ϕ in Eq.(27), the metric coefficients A , B and C turn out to be

$$A = B = c_2^{\frac{1}{3}} (nlt + k_1)^{\frac{1}{n}} \exp\left[\frac{\lambda(nlt + k_1)^{\frac{n-1}{n}}}{3b(n-1)}\right], \quad n \neq 1 \quad (35)$$

$$C = c_2^{-\frac{2}{3}} (nlt + k_1)^{\frac{1}{n}} \exp\left[\frac{\lambda(nlt + k_1)^{\frac{n-1}{n}}}{3b(n-1)}\right], \quad n \neq 1. \quad (36)$$

The directional Hubble parameters H_i ($i = 1, 2, 3$) take the form

$$H_1 = H_2 = \frac{l}{nlt + k_1} + \frac{l\lambda}{3b(nlt + k_1)^{\frac{1}{n}}}, \quad (37)$$

$$H_3 = \frac{l}{nlt + k_1} - \frac{2l\lambda}{3b(nlt + k_1)^{\frac{1}{n}}}. \quad (38)$$

The mean generalized Hubble parameter becomes

$$H = \frac{l}{nlt + k_1} \quad (39)$$

while the volume scale factor turns out to be

$$V = (nlt + k_1)^{\frac{3}{n}}. \quad (40)$$

The expansion scalar θ and shear scalar σ take the form

$$\theta = \frac{3l}{nlt + k_1}, \quad (41)$$

$$\sigma^2 = \frac{\lambda^2 l^2}{3b^2(nlt + k_1)^{\frac{2}{n}}} \quad (42)$$

The mean anisotropy parameter A_m becomes

$$A_m = \frac{2\lambda^2}{9b^2}(nlt + k_1)^{2-\frac{2}{n}}. \quad (43)$$

Moreover, Eqs.(21)-(23) take the form

$$\frac{8\pi\rho}{\phi} = -l^2(3+2\varpi)(nlt+k_1)^{-2} - \left[\frac{\lambda^2 l^2}{3b^2} + \frac{m^2}{c_2^{\frac{2}{3}}} \exp\left(\frac{-2\lambda(nlt+k_1)^{\frac{n-1}{n}}}{3b(n-1)}\right) \right] (nlt+k_1)^{-\frac{2}{n}}, \quad (44)$$

$$-\frac{8\pi\omega\rho}{\phi} = l^2[(3+2n+2\varpi)(nlt+k_1)^{-2} + \frac{\lambda^2}{3b^2}(nlt+k_1)^{-\frac{2}{n}}], \quad (45)$$

$$-\frac{8\pi(\omega+\delta)\rho}{\phi} = l^2(3+2n+2\varpi)(nlt+k_1)^{-2} + \left[\frac{\lambda^2 l^2}{3b^2} - \frac{m^2}{c_2^{\frac{2}{3}}} \exp\left(\frac{-2\lambda(nlt+k_1)^{\frac{n-1}{n}}}{3b(n-1)}\right) \right] (nlt+k_1)^{-\frac{2}{n}}. \quad (46)$$

3.2 Dark Energy Model of the Universe when $n = 0$.

The average scale factor for this model of the universe is $a = k_2 \exp(lt)$ and hence ϕ takes the form

$$\phi = \frac{b}{k_2^2} \exp(-2lt). \quad (47)$$

Inserting this value of ϕ in Eq.(27), the metric coefficients A , B and C become

$$A = B = c_2^{\frac{1}{3}} k_2 \exp(lt) \exp\left[-\frac{\lambda \exp(-lt)}{3b l k_2}\right], \quad (48)$$

$$C = c_2^{-\frac{2}{3}} k_2 \exp(lt) \exp\left[\frac{2\lambda \exp(-lt)}{3b l k_2}\right]. \quad (49)$$

The directional Hubble parameters H_i become

$$H_1 = H_2 = l + \frac{\lambda \exp(-lt)}{3b k_2}, \quad (50)$$

$$H_3 = l - \frac{2\lambda \exp(-lt)}{3b k_2}, \quad (51)$$

while the mean generalized Hubble parameter and the volume scale factor turn out to be

$$H = l, \quad V = k_2^3 \exp(3lt). \quad (52)$$

The expansion scalar θ and shear scalar σ take the form

$$\theta = 3l, \quad \sigma^2 = \frac{\lambda^2 \exp(-2lt)}{3b^2 k_2^2}. \quad (53)$$

The mean anisotropy parameter A_m for this model become

$$A_m = \frac{2\lambda^2 \exp(-2lt)}{9b^2 l^2 k_2^2}. \quad (54)$$

For this exponential model of universe, Eqs.(21)-(23) take the form

$$\frac{8\pi\rho}{\phi} = -l^2(3+2\varpi) - \frac{\lambda^2 \exp(-2lt)}{3b^2 k_2^2} - \frac{m^2}{c_2^{\frac{2}{3}} k_2^2} \exp(-2lt + \frac{2\lambda \exp(-lt)}{3bl k_2}), \quad (55)$$

$$- \frac{8\pi\omega\rho}{\phi} = l^2(3+2\varpi) + \frac{\lambda^2 \exp(-2lt)}{3b^2 k_2^2}, \quad (56)$$

$$- \frac{8\pi(\omega + \delta)\rho}{\phi} = l^2(3+2\varpi) + \frac{\lambda^2 \exp(-2lt)}{3b^2 k_2^2} - \frac{m^2}{c_2^{\frac{2}{3}} k_2^2} \exp(-2lt + \frac{2\lambda \exp(-lt)}{3bl k_2}). \quad (57)$$

4 Singularity Analysis

The Riemann tensor is a useful tool to determine whether a singularity is essential or coordinate. If the curvature becomes infinite at a certain point, then the singularity is essential. We can construct different scalars from the Riemann tensor and thus it can be verified whether they become infinite somewhere or not. Infinite many scalars can be constructed from the Riemann tensor, however, symmetry considerations can be used to show that there are only a finite number of independent scalars. All others can be expressed in terms of these. In a four-dimensional Riemann spacetime, there are only 14 independent curvature invariants. Some of these are

$$R_1 = R = g^{ab} R_{ab}, \quad R_2 = R_{ab} R^{ab}, \quad R_3 = R_{abcd} R^{abcd}, \quad R_4 = R_{cd}^{ab} R_{ab}^{cd}.$$

Here we give the analysis for the first invariant commonly known as the Ricci scalar for both models.

For model of universe with power law expansion, we can write Ricci scalar

$$R = -2\left[\frac{l^2(6-3n)}{a^{2n}} + \frac{\lambda^2 l^2}{3b^2 a^2} - \frac{m^2}{c_2^{\frac{2}{3}} a^2 \exp(\frac{2\lambda a^{n-1}}{3b(n-1)})}\right], \quad (58)$$

while for exponential model, it is given by

$$R = -2\left[6l^2 + \frac{\lambda^2}{3b^2 a^2} - \frac{m^2}{c_2^{\frac{2}{3}} a^2 \exp(\frac{2\lambda a}{3bl})}\right]. \quad (59)$$

Both of these models show that singularity occurs at $a = 0$.

5 Concluding Remarks

This paper is devoted to explore the solutions of Bianchi type *III* cosmological models in Brans Dicke theory of gravitation in the background of anisotropic dark energy. We use the power law relation between ϕ and a to find the solution. The assumption of constant deceleration parameter leads to two models of universe, i.e. power law model and exponential model. Some important cosmological physical parameters for the solutions such as expansion scalar θ , shear scalar σ^2 , mean anisotropy parameter and average Hubble parameter are evaluated.

First we discuss power law model of the universe. This model corresponds to $n \neq 0$ with average scale factor $a = (nlt + k_1)^{\frac{1}{n}}$. It has a point singularity at $t \equiv t_s = -\frac{k_1}{nl}$. The physical parameters H_1 , H_2 , H_3 and H are all infinite at this point but the volume scale factor vanishes here. The metric functions A , B and C vanish at this point of singularity. Thus, it is concluded from these observations that the model starts its expansion with zero volume at $t = t_s$ and it continues to expand for $0 < n < 1$.

The exponential model of the universe corresponds to $n = 0$ with average scale factor $a = k_2 \exp(lt)$. It is non-singular because exponential function is never zero and hence there does not exist any physical singularity for this model. The physical parameters H_1 , H_2 , H_3 are all finite for all finite values of t . The mean generalized Hubble parameter H is constant while metric functions A , B and C do not vanish for this model. The volume scale factor increases exponentially with time which indicates that the universe starts its expansion with zero volume from infinite past.

The isotropy condition, i.e., $\frac{\sigma^2}{\theta} \rightarrow 0$ as $t \rightarrow \infty$, is also satisfied in each case. It is mentioned here that the behavior of these physical parameters is consistent with the results already obtained in GR [7]. The variable EoS parameter ω for both models turn out to be

$$\omega = \frac{l^2[(3 + 2n + 2\varpi)(nlt + k_1)^{-2} + \frac{\lambda^2}{3b^2}(nlt + k_1)^{-\frac{2}{n}}]}{l^2(3 + 2\varpi)(nlt + k_1)^{-2} + [\frac{\lambda^2 l^2}{3b^2} + \frac{m^2}{c_2^{\frac{2}{3}}} \exp(\frac{-2\lambda(nlt+k_1)^{\frac{n-1}{n}}}{3b(n-1)})](nlt + k_1)^{-\frac{2}{n}}},$$

$$\omega = \frac{l^2(3 + 2\varpi) + \frac{\lambda^2 \exp(-2lt)}{3b^2 k_2^2}}{l^2(3 + 2\varpi) + \frac{\lambda^2 \exp(-2lt)}{3b^2 k_2^2} + \frac{m^2}{c_2^{\frac{2}{3}} k_2^2} \exp(-2lt + \frac{2\lambda \exp(-lt)}{3b l k_2})}.$$

Both of these equations suggest that at $t = 0$, ω has a positive value which indicates that the universe was matter dominated in its early phase of its existence. At $t \rightarrow \infty$, the value of ω turns out to be zero which indicate that the pressure of the universe vanishes at that epoch.

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